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ABOUT TRAVELLING WAVES IN FERMI-PASTA-ULAM SYSTEM ON 2D-LATTICE

We study the Fermi–Pasta–Ulam system that describes the dynamics of an infinite system of nonlinearly coupled particles on two dimensional lattice

$$\ddot{q}_{n,m}(t) = W_1'(q_{n+1,m}(t) - q_{n,m}(t)) - W_1'(q_{n,m}(t) - q_{n-1,m}(t)) +$$
$$+ W_2'(q_{n,m+1}(t) - q_{n,m}(t)) - W_2'(q_{n,m}(t) - q_{n,m-1}(t)), \ (n,m) \in \mathbb{Z}^2, \quad (1)$$

where $q_{n,m} = q_{n,m}(t)$ is a coordinate of (n,m)-th particle at time t, $W_1, W_2 \in C^1(\mathbb{R})$ are the potentials of neighbor interactions.

Travelling wave is a solution of the form

$$q_{n,m}(t) = u(n \cos\varphi + m \sin\varphi - ct),$$

where the function $u(s)$, $s = n \cos\varphi + m \sin\varphi - ct$, is called the profile function, or simply profile, of the wave and the constant $c \neq 0$ the speed of the wave. Making use the travelling wave we obtain the equation

$$c^2 u''(s) = W_1'(u(s + \cos\varphi) - u(s)) - W_1'(u(s) - u(s - \cos\varphi)) +$$
$$+ W_2'(u(s + \sin\varphi) - u(s)) - W_2'(u(s) - u(s - \sin\varphi)), \quad (2)$$

for the profile function $u(s)$. This equation has, actually, a variational structure.

We obtain, by means of the critical points method, a results on the existence of nonconstant periodic and solitary travelling waves.

1. Bak S. M. Existence of periodic traveling waves in Fermi-Pasta-Ulam system on 2D-lattice, *Matematychni Studii*, **37** (2012), 76–88. (in Ukrainian)

2. Bak S. M., Kovtonyuk G. M. Existence of solitary traveling waves in Fermi-Pasta-Ulam system on 2D lattice, *Matematychni Studii*, **50** (2018), 75–87.