ABOUT HETEROCLINIC TRAVELLING WAVES IN DISCRETE SINE–GORDON TYPE EQUATION

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We study the discrete sine-Gordon equation (DSG) that describes the dynamics of an infinite system of nonlinearly coupled nonlinear oscillators on two-dimensional lattice

$$\ddot{q}_{n,m} = V'(q_{n+1,m} - q_{n,m}) - V'(q_{n,m} - q_{n-1,m}) + V'(q_{n,m+1} - q_{n,m}) -$$
$$-V'(q_{n,m} - q_{n,m-1}) - K\sin(q_{n,m}), \quad (n,m) \in \mathbb{Z}^2, K > 0,$$

where $q_{n,m}$ be a generalized coordinate of the (n,m)-th oscillator at the time t. Notice that this system is a representative of a wide class of systems called lattice dynamical systems extensively studied in last decades. A comprehensive presentation of results about travelling waves for 1D Fermi-Pasta-Ulam (FPU) lattices is given in [17]. The existence of periodic travelling waves in FPU system on 2D-lattice is studied in [5]. In [13] certain results of such type are obtained by means of bifurcation theory, while in [1] and [2] the existence of periodic and solitary travelling waves is studied by means of the critical point theory. In papers [3], [4], [11], [12] travelling waves for systems of linearly coupled oscillators on 2D–lattice are studied. Paper [16] is devoted to periodic and homoclinic travelling waves for one-dimensional chain of nonlinearly coupled nonlinear particles. In [7] and [8] it is obtained a results on the existence of periodic traveling waves for the system of nonlinearly coupled nonlinear oscillators on 2D-lattice. In paper [14] contains a result on the existence of heteroclinic travelling waves for the DSG with linear interaction, while in [15] periodic, homoclinic and heteroclinic travelling waves for such systems with nonlinear interaction are studied. In paper [6] it is obtained a result on the existence of periodic travelling waves for DSG with nonlinear interaction on 2D-lattice, while [9] is devoted to the existence of heteroclinic travelling waves for DSG with linear interaction on 2D-lattice.

A travelling wave solution is a function of the form $q_{n,m}(t) = u(n\cos\varphi + m\sin\varphi - ct)$, where the profile function $u(s)$ satisfies the equation

$$c^2u''(s) = V'(u(s+\cos\varphi) - u(s)) - V'(u(s) - u(s-\cos\varphi)) +$$
$$+V'(u(s+\sin\varphi) - u(s)) - V'(u(s) - u(s-\sin\varphi)) - K\sin(u(s)). \quad (1)$$

The constant $c \neq 0$ is called the speed of the wave. An important role is played by the quantity c_1 defined by the equation $c_1^2 := 2\sup_{|r|<6\pi}\left|\frac{V(r)}{r^2}\right|$. The profile function of heteroclinic travelling wave satisfies the following conditions

$$\lim_{s\to-\infty} u(s) = -\pi \quad \text{and} \quad \lim_{s\to+\infty} u(s) = \pi. \tag{2}$$

We assume

(i) $V(r) \in C^1(\mathbb{R})$, $V(0) = 0$ *and* $V(r) \geq 0$ *for all* $r \in \mathbb{R}$;

(ii) $\lim\limits_{r\to\pm\infty} V(r) = +\infty$;

(iii) there exists finite $\lim\limits_{r\to 0} \left|\frac{V(r)}{r^2}\right|$;

(iv) the wave speed c *satisfies* $c^2 > c_1^2$.

The following theorem is the main result of the paper.

Theorem 1. *Assume (i)–(iv). Suppose that* c *is large enough to ensure* $\delta < \pi$ *for* $\delta := \frac{4c_1^2}{c^2 - c_1^2 + c\sqrt{c^2 - c_1^2}}$. *Then Eq. (1) has a solution* u *that satisfies the boundary conditions (2).*

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