

ON EXPONENTIAL DECAY OF SOLITARY TRAVELING
WAVE SOLUTIONS TO THE FERMI-PASTA-ULAM-TYPE
SYSTEMS ON 2D LATTICE

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We consider the Fermi-Pasta-Ulam-type systems that describe the dynamics of an infinite systems of nonlinearly coupled identical particles on a two dimensional lattice:

$$\ddot{q}_{n,m} = W_1'(q_{n+1,m} - q_{n,m}) - W_1'(q_{n,m} - q_{n-1,m}) + \\ + W_2'(q_{n,m+1} - q_{n,m}) - W_2'(q_{n,m} - q_{n,m-1}), \quad (n, m) \in \mathbb{Z}^2, \quad (1)$$

where $q_{n,m} = q_{n,m}(t)$ is a coordinate of the (n, m) -th particle at time t , W_1 and W_2 are the potentials of interaction.

A traveling wave solution of Eq. (1) is a function of the form

$$q_{n,m}(t) = u(n \cos \varphi + m \sin \varphi - ct). \quad (2)$$

We consider solitary traveling wave solutions, whose profile $u(s)$ satisfies the following condition:

$$\lim_{s \rightarrow \pm\infty} u(s) = u(\pm\infty) = 0. \quad (3)$$

By the Paley-Wiener Theorem, the exponential estimate for the profiles of such solutions is obtained.

1. Bak S.M., Kovtonyuk G.M. *Existence of traveling waves in Fermi–Pasta–Ulam type systems on 2D–lattice*. J. Math. Sci., 2021, **252** (4), 453-462.
2. Bak S.M., Kovtoniuk H.M. *Exponential decay of solitary traveling wave solutions to the Fermi-Pasta-Ulam-type systems on 2D lattice*. Ukr. Math. Bull. 2025.